Exponent Rules

Product Rule

If $a \neq 0$, and m and n are real numbers, then

$$a^m \cdot a^n = a^{m+n}$$

Multiplying with same base, keep the base and add the exponents

Ex:
$$x^5 \cdot x^7 = x^{5+7} = x^{12}$$

Power-to-Power Rule

If $a \neq 0$, and m and n are real numbers, then

$$(a^m)^n = a^{m \cdot n}$$

A base to a power, to another power, keep the base and multiply the exponents

Ex:
$$(x^5)^3 = x^{5 \cdot 3} = x^{15}$$

Product-to-Power Rule

If $a \neq 0$ and $b \neq 0$, and m is a real number, then

$$(ab)^m = a^m b^m$$

When a product is raised to a power, raise each factor to the power

Ex:
$$(xy)^3 = x^3y^3$$

Zero Exponent

If $\alpha \neq 0$, then

$$a^0 = 1$$

Any nonzero number to the zero power equals 1.

Ex:
$$(5x)^0 = 1$$

Ex:
$$5x^0 = 5 \cdot x^0 = 5 \cdot 1 = 5$$

Quotient Rule

If $a \neq 0$, and m and n are real numbers, then

$$\frac{a^m}{a^n} = a^{m-n}$$

Dividing with same base, keep the base and subtract the exponents

Ex:
$$\frac{x^{12}}{x^5} = x^{12-5} = x^7$$

Quotient-to-Power Rule

If $a \neq 0$ and $b \neq 0$, and m is a real number, then

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Quotient to power, raise both numerator and denominator to power

Ex:
$$\left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$$

Negative Exponent

If $a \neq 0$, and m is a positive real number, then

$$a^{-m} = \frac{1}{a^m} \quad \text{and} \quad \frac{1}{a^{-m}} = a^m$$

Base to negative power moves from numerator to denominator, or vice versa, the sign of power changes.

Ex:
$$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

Ex:
$$\frac{1}{x^{-5}} = x^5$$