Steps To Graph Rational Functions

1. **Make sure the numerator and denominator of the function are arranged in descending order of power.**

2. **Find the domain**

   a. Factor the denominator of the function, completely.

   b. Find the real zeros of the denominator by setting the factors equal to zero and solving.

   c. Write "Domain $= \{x | x \neq$ _____ $\}$"

3. **Factor the numerator and denominator, completely. Cancel any common factors between the numerator and denominator. (After you have canceled factors, you are left with the <u>reduced</u> function. Use the <u>reduced</u> function for all remaining steps.) If there are no common factors, then skip Step 4.**

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   4. Find any Holes

   a. Set any common factor found in Step 3 equal to zero and solve.

       This is the x-**coordinate** for the hole.

   b. Substitute this x-value back into the <u>reduced</u> function found in Step 3.

       This is the y-**coordinate** for the hole.

   b. Plot the hole on the graph as an open circle.

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5. **Find any Vertical Asymptotes**

   a. Set each factor from the denominator of the <u>reduced</u> function equal to zero and solve.

   b. Write each equation in the form $x =$____ .

   c. Draw each vertical asymptote as a dotted line on the graph.

6. **Find the Horizontal Asymptote (or Slant Asymptote)**

   Refer to the <u>reduced</u> function. You will need to know the leading terms from the numerator and the denominator. That is,

   $$F(x) = \frac{a_n x^n + \ldots}{b_m x^m + \ldots},$$

   where the **degree of the numerator is** n and the **degree of the denominator is** m.

   i. If $n < m$, then the x-**axis** (given by $y = 0$) is the horizontal asymptote of the graph.

   ii. If $n = m$, then the line $y = \dfrac{a_n}{b_m}$ is the horizontal asymptote of the graph.

   iii. If $n > m$, then the graph has **no** horizontal asymptote.

   The graph will only have a **slant asymptote** if $n = m + 1$. To find the slant asymptote, use long division. The **slant asymptote** will be the line given by $y =$ **quotient.**

$$\text{divisor} \overline{) \ \text{dividend}}^{\textstyle \text{quotient}}$$

7. Find any x-intercepts

    a.   Set the factors of the numerator from the <u>reduced</u> function equal to zero and solve.

    b.   Write the x-intercepts in the form (__ , 0).

    c.   Plot these points.

8. Find any y-intercepts

    a.   Substitute zero everywhere x appears in the <u>reduced</u> function and simplify.
        (*Hint:* This is just the ratio of the constants from the numerator and denominator.)

    b.   Write the y-intercept in the form (0, __).

    c.   Plot this point.

(NOTE: If you end up with division by zero during Step 8, then there is **no** y-intercept.)

9. Find test points

Make a T-chart with a few more points to the right and left of each vertical asymptote.

x	$f(x)$

◇ Refer to the vertical asymptotes you found in Step 5. When adding more points to your graph, remember that one vertical asymptote creates two branches of the graph, two vertical asymptotes creates three branches, and so on. So, you must find additional points that lie on either side of each vertical asymptote.

◇ Avoid simple arithmetic mistakes when you substitute x values into the function. Use a calculator, if necessary, and pay attention to signs.

◇ Sketch your curves so that they follow the asymptotes, without adding any extra curlicues to the graph.

◇ **The curves will never cross any vertical asymptote but they may cross a horizontal asymptote.**