Trigonometric Identities

Double - Angle Formulas

$$\sin(2u) = 2\sin(u)\cos(u)$$

$$\cos(2u) = \cos^2(u) - \sin^2(u) = 2\cos^2(u) - 1 = 1 - \sin^2(u)$$

$$\tan(2u) = \frac{2\tan(u)}{1 - \tan^2(u)}$$

Half - Angle Formulas

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}$$

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos(u)}{\sin(u)} = \frac{\sin(u)}{1+\cos(u)}$$

The signs of $\sin(u/2)$ and $\cos(u/2)$ depend on the quadrant in which $(u/2)$ lies.

Pythagorean Identities

$$\sin^2(u) + \cos^2(u) = 1$$

$$1 + \tan^2(u) = \sec^2(u) \quad 1 + \cot^2(u) = \csc^2(u)$$

Negative Angle Identities

$$\sin(-u) = -\sin(u) \qquad \csc(-u) = -\csc(u)$$

$$\cos(-u) = \cos(u) \qquad \sec(-u) = \sec(u)$$

$$\tan(-u) = -\tan(u) \qquad \cot(-u) = -\cot(u)$$

Cofuntion Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos(u) \quad \csc\left(\frac{\pi}{2} - u\right) = \sec(u)$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin(u) \quad \sec\left(\frac{\pi}{2} - u\right) = \csc(u)$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot(u) \quad \cot\left(\frac{\pi}{2} - u\right) = \tan(u)$$

Power - Reducing Formulas

$$\sin^2(u) = \frac{1-\cos(2u)}{2} \quad \csc^2(u) = \frac{2}{1-\cos(2u)}$$

$$\cos^2(u) = \frac{1+\cos(2u)}{2} \quad \sec^2(u) = \frac{2}{1+\cos(2u)}$$

$$\tan^2(u) = \frac{1-\cos(2u)}{1+\cos(2u)} \quad \cot^2(u) = \frac{1+\cos(2u)}{1-\cos(2u)}$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin(u)\cos(v) \pm \cos(u)\sin(v)$$

$$\cos(u \pm v) = \cos(u)\cos(v) \mp \sin(u)\sin(v)$$

$$\tan(u \pm v) = \frac{\tan(u) \pm \tan(v)}{1 \mp \tan(u)\tan(v)}$$

Product - to - Sum Formulas

$$\sin(u)\sin(v) = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\cos(u)\cos(v) = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin(u)\cos(v) = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos(u)\sin(v) = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

Sum - to - Product Formulas

$$\sin(u) \pm \sin(v) = 2\sin\left(\frac{u \pm v}{2}\right)\cos\left(\frac{u \mp v}{2}\right)$$

$$\cos(u) + \cos(v) = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos(u) - \cos(v) = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\sin(\cos^{-1}(x)) = \sqrt{1-x^2}$$

$$\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$$

$$\sec(\tan^{-1}(x)) = \sqrt{1+x^2}$$

$$\tan(\sec^{-1}(x)) = \begin{cases} \sqrt{x^2-1}\ , & x \geq 1 \\ -\sqrt{x^2-1}\ , & x \leq -1 \end{cases}$$

Law of Sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$