

MATHEMATICS OF FINANCE

Changing a Fraction to a Percent

Express $\frac{3}{40}$ as a percent.

$$3 \div 40 = 0.075$$
$$0.075 = 7.5\%$$

To express a fraction as a percent, divide the numerator by the denominator, multiply by 100 to move the decimal point in the result two places to the right, and add a percent sign.

Changing a Decimal to a Percent

Express .96 as a percent

$$0.96 = 96\%$$

To express a decimal as a percent, multiply by 100 to move the decimal point two places to the right, and add a percent sign.

Changing a Percent to a Decimal

Express 130% as a decimal

$$130\% = 1.3$$

To express a percent as a decimal, divide by 100 to move the decimal point two places to the left, and remove the percent sign.

Finding the Total Price Including Tax

San Antonio's current sales tax rate is 8.25%. If a store is selling a graphing calculator for \$109.99, how much tax is paid? What is the graphing calculator's total cost?

$$\begin{aligned}\text{Sales Tax Amount} &= .0825 \cdot \$109.99 \\ &= \$9.07 \\ \\ \text{Total Price} &= \$109.99 + \$9.07 \\ &= \$119.06\end{aligned}$$

$$\begin{aligned}\text{Sales Tax Amount} &= \text{Tax Rate} \cdot \text{Original Price} \\ \text{Total Price} &= \text{Original Price} + \text{Sales Tax Amount}\end{aligned}$$

Finding the Total Discounted Price

Hand sanitizer that normally sells for \$5.79 is now on sale at 40% off. What is the hand sanitizer's discounted price?

$$\begin{aligned}\text{Discount Amount} &= .40 \cdot \$5.79 \\ &= \$2.32 \\ \\ \text{Discounted Price} &= \$5.79 - \$2.32 \\ &= \$3.47\end{aligned}$$

$$\begin{aligned}\text{Discount Amount} &= \text{Discount Rate} \cdot \text{Original Price} \\ \text{Discounted Price} &= \text{Original Price} - \text{Discount Amount}\end{aligned}$$

Showing Percent Increase/Decrease

An 8GB Reading Tablet normally sells for \$139.00. The discounted price is \$119.00. What is the percent decrease of the tablet?

$$\begin{aligned}\text{Percent Change} &= \frac{\$139.00 - \$119.00}{\$139.00} \\ \text{Percent Change} &= \frac{\$20.00}{\$139.00} \\ \text{Percent Change} &= .144 \\ \text{Percent Change} &= 14.4\%\end{aligned}$$

$$\text{Percent Change} = \frac{\text{Original Amount} - \text{New Amount}}{\text{Original Amount}}$$

Positive Percent Change is a Percent Decrease
Negative Percent Change is a Percent Increase

Calculating Simple Interest

You deposit \$1500 in a savings account. Your bank provides a 0.01% rate for savings accounts. Find the interest earned after 1 year.

$$\begin{aligned}I &= \$1500 \cdot .0001 \cdot 1 \\ I &= \$0.15\end{aligned}$$

$$I = PV \cdot r \cdot t$$

Using the Future Value with Simple Interest Formula

You decide to buy a certificate of deposit (CD) from your local bank. The two year CD offers a rate of 0.20%. How much must you put in the CD to have \$3000 in two years?

$$\begin{aligned}\$3000 &= PV(1 + .0020 \cdot 2) \\ \$3000 &= PV(1.0040) \\ \frac{\$3000}{1.0040} &= PV \\ \$2988.05 &= PV\end{aligned}$$

$$FV = PV(1 + r \cdot t)$$

$$FV = PV + PV \cdot r \cdot t$$

Using the Future Value with Compound Interest Formula (Compounded m Times per Year)

You deposit \$1500 in a savings account. Your bank provides a 0.01% annual rate for savings accounts. The interest is compounded monthly. Find the value of the account after one year.

$$FV = \$1500 \left(1 + \frac{.0001}{12} \right)^{12 \cdot 1}$$

$$FV = \$1500.15$$

$$FV = PV \left(1 + \frac{r}{m} \right)^{m \cdot t} \qquad FV = PV(1 + i)^n$$

Using the Future Value with Compound Interest Formula (Compounded Continuously)

Parents wish to have \$80,000 available for their child's education. If their child is three years old, how much must be set aside at 5% compounded continuously to meet their goal when the child is 18?

$$\$80,000 = PV \cdot e^{.05 \cdot (18-3)}$$

$$\frac{\$80,000}{e^{.05 \cdot (18-3)}} = PV$$

$$\$37,789.33 = PV$$

$$FV = PV \cdot e^{r \cdot t}$$

Comparing Different Interest Compounding Periods (Using the Annual Percentage Yield [APY])

What is the APY of an account paying 8% compounded quarterly?

$$APY = \left(1 + \frac{.08}{4} \right)^4 - 1$$

$$APY \approx 8.24\%$$

$$APY = \left(1 + \frac{r}{m} \right)^m - 1$$

Calculating the Future Value of an Annuity (Present Payments)

Suppose when you are 30, you decide to save for retirement by depositing \$4,000 into a Roth IRA at the end of each year. If the interest rate is 6.5% compounded annually, How much will you have from the Roth IRA after 35 years? How much is from interest?

$$FV = \frac{4000 \cdot \left[\left(1 + \frac{.065}{1} \right)^{1 \cdot 35} - 1 \right]}{\left(\frac{.065}{1} \right)}$$

$$FV = \$496,138.76$$

$$FV = \frac{PMT \cdot \left[\left(1 + \frac{r}{m} \right)^{m \cdot t} - 1 \right]}{\left(\frac{r}{m} \right)}$$

$$I = \$496,138.76 - \$4,000(35)$$

$$I = \$356,138.76$$

Calculating a Sinking Fund Payment

Parents wish to have \$80,000 available for their child's education. If their child is three years old, how much should they deposit per month at 5% compounded monthly to meet their goal when the child is 18?

$$PMT = \frac{\$80,000 \cdot \left(\frac{.05}{12} \right)}{\left[\left(1 + \frac{.05}{12} \right)^{12 \cdot 15} - 1 \right]}$$

$$PMT = \$299.31$$

$$PMT = \frac{FV \cdot \left(\frac{r}{m} \right)}{\left[\left(1 + \frac{r}{m} \right)^{m \cdot t} - 1 \right]}$$

Calculating the Present Value of an Annuity (Future Withdraws)

How much should you deposit in an account paying 8% compounded weekly in order to receive weekly payments of \$150 for the next 3 years?

$$PV = \frac{\$150 \cdot \left[1 - \left(1 + \frac{.08}{52} \right)^{-52 \cdot 3} \right]}{\left(\frac{.08}{52} \right)}$$

$$PV = \$20789.64$$

$$PV = \frac{PMT \cdot \left[1 - \left(1 + \frac{r}{m} \right)^{-m \cdot t} \right]}{\left(\frac{r}{m} \right)}$$

Calculating Fixed Installment Loan Payment (Amortization)

Suppose you decide to borrow \$30,000 for a new car. If you accept a 5 year loan at 4%, what are the monthly payments? What is the total interest?

$$PMT = \frac{\$30,000 \cdot \left(\frac{.04}{12} \right)}{\left[1 - \left(1 + \frac{.04}{12} \right)^{-12 \cdot 5} \right]}$$

$$PMT = \$552.50$$

$$PMT = \frac{PV \cdot \left(\frac{r}{m} \right)}{\left[1 - \left(1 + \frac{r}{m} \right)^{-m \cdot t} \right]}$$

$$I = \$552.50(60) - \$30,000$$

$$I = \$3,150$$