Divisibility Tests

Any number is **divisible by 2** if it is an even number. That is, if its last digit is 0, 2, 4, 6, or 8.

Any number is **divisible by 3** if the sum of its digits is divisible by 3. For example, 234 is divisible by 3 because $2 + 3 + 4 = 9$ and 9 is divisible by 3.

Any number is **divisible by 5** if the last digit is 0 or 5. For example, 250 and 255 are both divisible by 5.

Least Common Multiple

- Write the prime factorization of each number
- The LCM equals the product of each factor the highest number of times it repeats in any one factorization.

Adding, Subtracting, Multiplying and Dividing Fractions

To add (or subtract) fractions with the same denominators we add (or subtract) the numerators and keep the same denominator.

If $a, b, c \in W$, and $b \neq 0$, then

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \qquad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

To multiply fractions, we multiply their numerators together and then multiply their denominators together.

If $a, b, c, d \in W$, $b \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

To divide fractions, we multiply the first number by the reciprocal of the second number.

If $a, b, c, d \in W$, $b \neq 0$, $c \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$$

Dividing with Zero

For all real numbers n, if $n \neq 0$, then

$\frac{0}{n} = 0$ and $\frac{n}{0}$ is undefined

Adding and Subtracting Fractions with Different Denominators

To add (or subtract) fractions with different denominators, we must change one or both fractions to equivalent fractions with common denominators.

Ex. Write as an equivalent fraction with a denominator of 32.

$\frac{2}{8} = \frac{?}{32}$ $\quad$ $\frac{5}{2} = \frac{?}{32}$ $\quad$ $\frac{15}{16} = \frac{?}{32}$ $\quad$ $\frac{14}{4} = \frac{?}{32}$

$\frac{2}{8} = \frac{2}{8} \cdot 1$ $\quad$ $\frac{5}{2} = \frac{5}{2} \cdot 1$ $\quad$ $\frac{15}{16} = \frac{15}{16} \cdot 1$ $\quad$ $\frac{14}{4} = \frac{14}{4} \cdot 1$

$= \frac{2}{8} \cdot \frac{4}{4}$ $\quad$ $= \frac{5}{2} \cdot \frac{16}{16}$ $\quad$ $= \frac{15}{16} \cdot \frac{2}{2}$ $\quad$ $= \frac{14}{4} \cdot \frac{8}{8}$

$= \frac{8}{32}$ $\quad$ $= \frac{80}{32}$ $\quad$ $= \frac{30}{32}$ $\quad$ $= \frac{112}{32}$

Adding, Subtracting, Multiplying, and Dividing Decimals

To add (or subtract) decimals, we line up the decimal points vertically so that we can add common place values. If necessary, insert zeros to have the same number of digits for both numbers.

To Multiply Decimals

- Take the numbers without the decimals and multiply them to get the digits of the product.
- Place the decimal point in the product so that the answer has the sum of the number of decimal places from each factor, inserting zero(s) if necessary.

To Divide Decimals

- Move the decimal point in the divisor to make it a whole number, if necessary.
- Move the decimal point in the dividend (numerator) the same number of places to the right as in step 1.
- Divide. Write the decimal point in the answer directly above the decimal point in the dividend.

Adding, Subtracting, Multiplying, and Dividing Real Numbers

Addition

To add two real numbers with the ***same sign***, add the numbers and attach the common sign.

To add two real numbers with ***different signs***, subtract the numbers and attach the sign of the larger number.

Subtraction

$a - b = a + (-b)$

1. Rewrite subtraction as addition.
2. Change the sign of the number after the minus sign.
3. Follow the rules of addition.

Multiplication

The product of two real numbers with the ***same sign*** is positive.
The product of two real numbers with ***different signs*** is negative.

Division

The quotient of two real numbers (denominator $\neq 0$) with the ***same sign*** is positive.
The quotient of two real numbers (denominator $\neq 0$) with the ***different signs*** is negative.

Properties of Real Numbers

Commutative Property of Addition
For all real numbers a and b,

$$a + b = b + a$$

Commutative Property of Multiplication
For all real numbers a and b,

$$a \cdot b = b \cdot a$$

Associative Property of Addition
For all real numbers a, b, and c,

$$(a + b) + c = a + (b + c)$$

Associative Property of Multiplication
For all real numbers a, b, and c,

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Distributive Property
For all real numbers a, b, and c,

$$a(b + c) = a \cdot b + a \cdot c$$

Identity Property of Addition
For all real numbers a,

$$0 + a = a + 0 = a$$

Note: 0 is the additive identity element

Identity Property of Multiplication
For all real numbers a,

$$1 \cdot a = a \cdot 1 = a$$

Note: 1 is the multiplicative identity element

Inverse Property of Addition
For all real numbers of a,

$$a + (-a) = 0$$

Note: a and ($-a$) are called additive inverses.

Inverse Property of Multiplication
For all real numbers $a, a \neq 0$,

$$a \cdot \frac{1}{a} = 1$$

Note: a and $\frac{1}{a}$ are called multiplicative inverses.

Order of Operations (PEMDAS)

Parentheses

Exponents, work left to right

Multiplication
Division
> Perform these operations working left to right.

Addition
Subtraction
> Perform these operations working left to right.