Quick Review Sheet - Math 0303

Roots and Radicals

Product Rule and Quotient Rule for Radicals

Let a and b be positive real numbers, and $n \in N$ with $n \geq 2$. Then,

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \text{ where } b \neq 0$$

Rational Exponents

If $n \in N$ such that $n \geq 2$ and $\sqrt[n]{a}$ is a real number, then

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

Radical Expression with a Rational Exponent $\frac{m}{n}$

Let m and n be positive integers with no common factors such that $n \geq 2$ and $\sqrt[n]{a}$ is a real number, then

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

For the rational exponent $\frac{m}{n}$, m is the power (or exponent), while n is the root (or index).

Solving Equations Containing Square Roots

1. Isolate a radical
2. Raise both sides of the equation by the index of the radical.
3. If a radical remains, repeat steps 1 and 2.
4. Solve the equation.
5. Check all solutions.

The Imaginary Unit *i*

The imaginary unit i is defined as $i = \sqrt{-1}$ such that $i^2 = -1$.

Let $b \in R$ with $b > 0$. Then define $\sqrt{-b} = i\sqrt{b}$

Quadratic Equation

A **quadratic equation** is an equation that can be written in the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers with $a \neq 0$.

The Square Root Property

If $x^2 = k$ for any real number k, then $x = -\sqrt{k}$ or $x = \sqrt{k}$. A common shorthand notation is $x = \pm\sqrt{k}$.

How to Solve Using the Square Root Property

1. Isolate the squared term on one side of the equation with the constant term on the other side.
2. Use the Square Root Property to solve for the variable.
3. Check by substituting the solution(s) into the original quadratic equation.

Ex.

$$2x^2 - 16 = 0$$
$$2x^2 = 16$$
$$x^2 = 8$$
$$x = -\sqrt{8} \; or \; x = \sqrt{8}$$
$$x = -2\sqrt{2} \; or \; x = \sqrt{2}$$
$$x = \pm\, 2\sqrt{2}$$

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To Complete the Square on $ax^2 + bx + c = 0$

1. If $a = 1$, proceed to step 2. If $a \neq 1$, divide through by the leading coefficient a.
2. Isolate the variable terms on one side of the equation, and the constant term on the other side.
3. Take half of the linear term's coefficient and square it. If $a = 1$, this is $(\frac{1}{2}b)^2$. Add this quantity to both sides of the equation.
4. Factor the perfect square trinomial as a binomial squared.
5. Solve using the Square Root Property.

Solving by Using the Quadratic Formula

1. Write the quadratic equation in standard form
$$\mathrm{ax}^2 + \mathrm{bx} + \mathrm{c} = 0$$
2. Identify *a, b,* and *c.*
3. Write the Quadratic Formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
4. Substitute the values of *a, b,* and *c* into the Quadratic Formula.
5. Simplify and state the solutions.

Discriminant

The **discriminant**, $b^2 - 4ac$, is the radicand of the Quadratic Formula and determines the type and number of solutions to the quadratic equation $ax^2 + bx + c = 0$.

1. If $b^2 - 4ac > 0$, there are *two real* solutions.
2. If $b^2 - 4ac < 0$, there are *two complex* solutions.
3. If $b^2 - 4ac = 0$, there are *one real* solutions.

Methods for Solving any Quadratic Equation

1. If the quadratic equation is *easily* factored, solve by factoring.
2. If there is *no* middle term (that is, *bx*), use the Square Root Property.
3. If the leading coefficient is 1 and the middle coefficient is *even*, or if when you divide through by a the middle coefficient becomes *even*, complete the square.
4. In remaining situations, use the Quadratic Formula.

Graphing Quadratic Equations

In the form $y = ax^2 + bx + c$

Orientation

If $a > 0$, the parabola faces up.

If $a < 0$, the parabola faces down.

Axis of Symmetry

The vertical line given by the equation $x = \frac{-b}{2a}$

Vertex

The vertex has the *x*-coordinate $x = \frac{-b}{2a}$. To find the y-coordinate of the vertex, substitute $x = \frac{-b}{2a}$ into the equation and find *y*.

Intercept

The *y*-intercept is given by (0,c).

Find the *x*-intercept(s)(if any) by setting *y = 0* and solving for *x*.

In the form $y = a(x - h)^2 + k$

Orientation

If $a > 0$, the parabola faces up.

If $a < 0$, the parabola faces down.

Vertex

The vertex is given by (*h,k*).

Axis of Symmetry

The vertical line given by *x = h*

Intercepts

Find the *x*- and *y*-intercepts, if applicable.

**For both methods, find additional points as needed using a table and the axis of symmetry.*